Ulrich Müther, in East Germany, popularized shells for public buildings and continued building shells up until the 1980s.

The breakthrough that reinforced concrete shells meant in the history of construction is related to the unsurpassed economy of material and resources needed to build shell roofs compared to those built with masonry of brickwork. A shell to cover a circular area of around 45 m of diameter, like Torroja's Algeciras market roof, could be built with just 8 cm thick reinforced concrete, whereas a masonry dome of similar diameter, like the Agrippa's Pantheon in Rome, required a much thicker structure. The low material consumption and the low cost of labor that was usually required to build the framework and cast the concrete, were key to the success of shells in the mid-20th century. The increase of labor costs was among the reasons why they practically ceased to be built.

Figure 1: Left: Los Manantiales shell in Xochimilco, Mexico D.F. Right: Restaurant shell at Oceanogràfic in Valencia.

In the mid-1990s, Félix Candela was invited to design the roof for the restaurant located in the Oceanogràfic, a new aquarium facility that was being planned in Valencia (Spain). Candela proposed to build a structure similar to the famous Los Manantiales restaurant shell roof in Xochimilco, Mexico D.F., designed and built by his company, Cubiertas Ala, in 1958. The original structure is an extremely lightweight groined vault with a basic thickness of only 4 cm (Fig. 1, left). Candela passed away in 1997, leaving the project in its very beginnings. The author, together with Alberto Domingo, was in charge of developing the basic and detailed structural design of the shell, which was finished in 2000 (Fig. 1, right). The article is organized as follows: Section 2 reviews Candela's design method; Section 3 describes the shell built at the Oceanogràfic in Valencia and the challenges faced for the design; finally, Section 4 summarizes the singular features of the structure.

2 Candela's shell design method

Félix Candela (1910-1997) was a Spanish-Mexican architect. He is considered one of the great masters of shell design and construction [2]. He was the co-founder and inspirational force of Cubiertas Ala, a design and construction company specialized in thin shell structures. The company was active during the 1950s and 60s and realized almost 900 shell projects. The majority of the projects were hyperbolic paraboloidal (hypar) shells [3].

There are several fundamental aspects to understand the attraction that the geometry of the hypar exerted on Candela:

- Thanks to their negative Gaussian curvature, when the boundary conditions are adequately designed, hypars transfer external loads mainly by means of membrane forces; bending and twisting moments are negligible. Furthermore, double curvature considerably reduces the risk of buckling, compared to single curvature shells.
- Hyperbolic paraboloids are doubly ruled surfaces, which greatly facilitates the construction of the formwork by means of straight beams and laths. In the words of Candela "…of all the shapes that we can give to a shell, the easiest and most practical to construct is the hyperbolic paraboloid." [4].
- Analyzing and determining membrane stress states in an hypar can be systematized for a variety of geometric parameters and loading conditions. Candela contributed to developing formulas applicable to a multiplicity of cases and trained the staff of his company to perform the calculations.

Candela's structural design procedure consisted of finding a state of membrane forces in the shell in equilibrium with the external forces by *partially prescribing force boundary conditions*. No compatibility of displacements was considered in the design.

Hyperbolic paraboloids are defined by the equation

$$z = k\,x\,y \tag{1}$$

where x and y are oblique axes forming an angle ω, z is a perpendicular axis and k is a constant that scales the geometry in the z direction. A peculiarity of the hypar geometry is that the equilibrium equation in the z direction directly provides the value of the membrane shear forces N_{xy} acting on the sides of a shell element oriented along the generatrixes (Fig. 2). The (oblique) membrane forces acting on these sides and directed along the opposite generatrix, N_{xx}, N_{yy}, can be obtained by integration; the resulting expressions contain arbitrary functions of integration. The details of the method are thoroughly explained in [4].

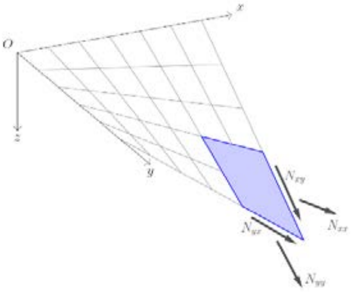

Figure 2: Membrane forces acting on an element of shell surface

Considering an hypar surface bounded by four straight edges, the shear forces along all boundary edges are directly obtained from given loads, as explained in the previous paragraph, and the oblique forces on two edges can be prescribed by means of defining suitable functions of integration. Once the integration functions are defined, the membrane state in the hypar is fully defined; the oblique forces on the two opposite edges will result from the calculations and can be understood as reaction forces. Fig. 3, left, shows a case in which the integration functions are such that the oblique forces along the two right-side edges are null, and reactions with tangent and normal components develop along the left-side edges. Therefore, there will be two edges along which only shear forces act, and the other two with shear plus normal forces. In terms of design, the edges with only shear can be equipped with a lighter edge beam, whereas the other two need a stronger beam to carry the forces to the foundation (Fig.3, right).

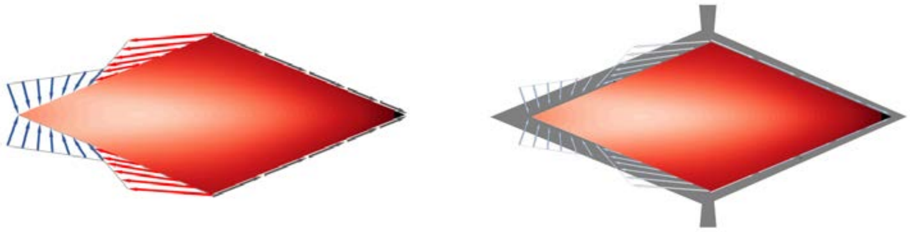

Figure 3: Boundary forces with prescribed zero-oblique forces on the rightmost edges (left); required edge beams on every edge (right)

Candela went a step further and showed that free edges resulting from cutting the paraboloid by inclined planes could be designed with zero-force boundary conditions. This fact led him to design shells with no edge beam and extreme lightweight appearance like *Los Manantiales* (Fig. 1, left).

Although not expressed in that way by Candela, his design procedure can be classified as a limit analysis method: finding a stress state in equilibrium with the external forces that complies with the yield criterion is a sufficient condition for the shell to be at worst at the collapse situation. The development of concrete standards in the 1970s enforced the consideration of compatibility of deformations in the design of shells, rendering Candela's method insufficient for the checks required by codes.

3 The shell at the *Oceanogràfic* in Valencia

The structure that is the subject of this text forms the roof of the restaurant located in the center of the *Oceanogràfic* aquarium park in Valencia. The shell is a 8-leaf groined vault formed by the intersection of four hypars with rotational symmetry that share a common downward-pointing vertical z axis. The angle ω between the oblique x and y axes is 22.5° and the hypar constant is $k = -0.0351\ \mathrm{m}^{-1}$ – see Eq. (1). The exterior boundary of the roof is created by trimming the hypars with inclined planes; this creates free edges with no edge beam that are characteristic of a number of Candela's structures. The roof rests on eight supports at the bases of the ribs resulting from the intersections of the hypar surfaces. The dimensions of the shell were adapted to the functional requirements of the facility, resulting in a span between opposite supports of 35.5 m and a perimetral separation between supports of 13.6 m. The height at the center of the vault is 8.1 m with respect to the support level. The diameter of the structure is about 5 m larger than the original *Los Manantiales* roof and the hypar leaves are steeper as well.

The main challenge in the design of Valencia's shell was to be faithful to the main features of Candela's structures: the design should be a reinforced concrete shell of the minimum possible thickness with the surface geometry described in the previous paragraph. Candela's shells were extremely thin (around 4 cm) and only had one layer of mesh reinforcement. As discussed in the previous section, only membrane forces were considered in the design, and no kinematic compatibility at the boundaries was enforced. The required thickness in Valencia's shell was strongly related to the concrete cover required by the Spanish standard for concrete structures EH-93 (still in force at that time), and to the need of either one or two layers of reinforcement.

Applying Candela's calculation method to Valencia's shell led to very low membrane stresses for a surface load modelling self-weight plus an overload of 1 kN/m (see [5] for a detailed account of the calculations). However, initial tests with finite element models showed that there were load cases for which bending moments in the shell were not negligible (albeit small). Increasing the thickness of the shell didn't bring any substantial reduction in the magnitude of the bending moments. The straightforward solution would have been to design the shell with two layers of reinforcement to resist the moments, which would have increased the shell thickness to at least 15 cm, far from the extreme slenderness of Candela's structures.

The decision was to use steel fibers in the concrete mix, in addition to just one layer of conventional reinforcement. Steel fibers provide additional tensile strength to concrete cross sections, the magnitude of which is sufficient to resist the design bending moments. Additionally, steel fibers smear the concrete cracking and increase the ductility of the material. Using fibers allowed to design a shell with a base thickness of just 6 cm. Checking of steel-fiber reinforced concrete sections required the use of specific standards. Details of the design can be found in [6].

Figure 4: The finished shell prior to building the cladding.

An additional challenge concerned the lack of experience of Spanish contractors to build shells in the late 1990s. While there is a remarkable collection of 1970s concrete thin shells in the region (see the book including reference [5]), they were built mostly by specialized companies that did no longer exist. The crucial aspect for the contractor was to find a concrete casting procedure that avoided the dangerous situation of workers standing on the formwork during the process. Shotcreting was the preferred solution: it could be performed directly on the shallower parts of the roof, or from a cherry picker platform on the steeper parts, and it was compatible with using steel fibers in the concrete mix.

The concrete of the shell has a characteristic strength of 30 MPa and the yield strength of the passive reinforcement is 500 MPa. Several laboratory tests were carried out to decide the dosage of fibers [7]. Galvanized steel fibers type Dramix RC-80/35-BN, with a dosage of 50 kg/m^3 were used. They are 35 mm long, with a diameter of 0.45 mm and stepped ends. Dry sprayed concrete was used and in-situ tests were performed to ensure the workability of the concrete mix with the required fiber dosage.

The base reinforcement is a Ø8 mm @15 cm mesh following the parabolas resulting from sections through vertical planes parallel to the bisectors of each paraboloid's x and y axes. The ribs at the intersection of hypar lobes have approximately triangular cross-section with decreasing height (0.8 m at the supports and 0.25 m at the central intersection). The thickness of the shell increases from the base value to meet the rib cross section. Additional reinforcement layers are placed in these transitions. The shell is supported on elastomeric pot bearings. Reactions are transferred to lower columns in the basement that are joined to the deck slab at the level of the supports. Rebar ties embedded in the slab resist the horizontal components of the reactions. The total weight of the structure is approximately 5 280 kN, covering a useful area of about 1 000 m^2.

Figure 5: Left: Formwork and partially placed reinforcement. Right: Shotcreting

The falsework and formwork were built during June and July 2000. The placement of the reinforcement took place in August and September. Concreting began in mid-September and was completed in November. In total, the shell was built in about six months. Reference [7] includes additional details on the construction process.

4 Final remarks

Candela's last shell in Valencia is an extraordinary structure for several reasons: it is a re-interpretation, but not a copy, of one of the architect's most famous structures; it was designed and built at a time when the methods extensively used by Candela were long outdated and the knowledge to build shells was practically lost, which forced to find new methods and solutions; the proposal of using steel fibers in the concrete mix made possible to check the structural performance according to codes and to build the shell in an efficient and economic manner. To the author's knowledge, this shell is one of the first examples of architectural shells built with steel-fiber reinforced concrete.

Although nowadays there are few newly built concrete shells, other kinds of surface structures like gridshells or lattice shells, as well as shells with alternative materials (masonry, tile vaults) are being built in many places. Shells constitute excellent representations of the paradigm of building with less material and less waste, and consequently, experimentation and research in this realm should be further promoted.

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